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**INSTITUTE FOR PLASMA RESEARCH**

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## ABSTRACT

It has been noted that coronal holes appear to be associated with regions of diverging magnetic field in the corona. We set out to test the hypothesis that coronal holes may be caused by an increased flow of energy into the solar wind resulting directly from this diverging magnetic field pattern. Simple models were devised to approximate the energy flow down into the transition region and up into the solar wind as a function of the temperature, density, and rate of field line divergence in the corona. By assuming the rate of mechanical energy influx into the corona to be constant, it was then possible to solve numerically for the coronal temperature and density as a function of the rate of field line divergence. The results of these calculations demonstrate that a diverging field pattern can, indeed, bring about reductions in the temperature and density at the base of the corona comparable to those observed in coronal holes.

## I. Introduction

The work of Burton (1968), Tousey et al. (1968), and Munro and Withbroe (1972) has established the existence of "holes" in the corona characterized by abnormally low densities and temperatures; and Krieger et al. (1973) have found that such coronal holes appear to be the source of high velocity, enhanced density streams in the solar wind as observed at the earth's orbit. It has further been noted by Altschuler et al. (1972) that coronal holes appear to be associated with regions of diverging magnetic field in the corona.

We have noted that one effect of a diverging magnetic field would be to lower the "throat" in the equivalent "Laval nozzle" which represents the mechanism by which subsonic coronal plasma is accelerated into the supersonic solar wind (Parker, 1963). Thus an increase in the divergence of the magnetic field will result in higher power input from the corona into the solar wind, so that there will be a correspondingly smaller return of power from the corona via the transition region to the chromosphere. This would result in a lower temperature gradient in the transition region and a lower temperature of the corona.

Our aim in this article is to test this proposed interpretation of the mechanism of coronal holes by calculating the properties of a simplified model.

## II. Model Used

The model we have adopted comprises the following parts: (a) an energy source that injects a certain constant flux of energy into the base of the corona; (b) an energy outflow from the corona due to conduction of heat downward into the transition region; (c) an energy outflow from the corona due to particles flowing out into the solar wind; and (d) an empirical relation between the temperature $T_c$ and density $N_c$ at the base of the corona (from here on the subscript c will refer to quantities evaluated at the base of the corona or, equivalently, the top of the transition region). We proceed to elaborate the model used for each of these parts.

(a) Energy Source

The source of the energy flux F injected at the base of the corona is presumably mechanical waves propagated up from the photosphere, but for our purposes the nature of the source is unimportant; we require only that such a source exist. In reality this energy is probably deposited over a finite range of height, but here we assume for simplicity that it is all dumped at the base of the corona.

(b) Energy Loss into Transition Region

We need a simple model that will give us the rate of heat loss into the transition region as a function of the temperature at the base of the corona. To arrive at such a model we note that the transition region is in hydrostatic equilibrium, so that

$$0 = -2N\mu g - \frac{d}{dz}(2NkT) \qquad (1)$$

where N is the electron density, T is the temperature, k is Boltzmann's constant, $\mu$ is the mean particle mass (which we take to be half the mass of a proton), g is the surface gravity of the sun, and z is the height above the base of the transition region. We assume that the temperature structure of the transition region is dictated by the downward heat flux $F_d$, which we assume to be constant throughout the transition region. Hence

$$F_d = aT^{5/2} \frac{dT}{dz} \tag{2}$$

where $a = 1.0 \times 10^{-6}$ (in cgs units). Equations (1) and (2) can be combined to yield

$$NT = N_b T_b \exp \left\{ - \frac{2\mu ga}{5kF_d} \cdot \left( T^{5/2} - T_b^{5/2} \right) \right\} \tag{3}$$

where the subscript b refers to quantities evaluated at the base of the transition region. Since $T_b << T$ once we get much above the base of the transition region, equation (3) tells us that

$$NT \approx N_b T_b \exp \left\{ - \frac{2\mu ga}{5kF_d} \cdot T^{5/2} \right\} . \tag{4}$$

Equation (4) can be rewritten in the form

$$NT \approx N_b T_b \exp \left\{ - (T/T_F)^{5/2} \right\} \tag{5}$$

where

$$T_F \equiv \left( \frac{5kF_d}{2\mu ga} \right)^{2/5} . \tag{6}$$

By comparing the plot given in Figure 1a of the observed density versus temperature for the upper transition region and corona, based on data from Allen (1973), with the plot given in Figure 1b based on equation (5), we see that equation (5) best reproduces the observational data if $T_c$ is approximately equal to $T_F$. Hence, inverting equation (6), we obtain

$$F_d \approx \frac{2\mu ga}{5k} \cdot T_c^{5/2} \tag{7}$$

as our final equation giving the downward heat flux $F_d$ as a function of the temperature at the base of the corona.

(c) Energy Loss into Solar Wind

To calculate the energy loss into the solar wind, we wish to use the simplest model possible. We therefore assume the corona to be isothermal and in hydrostatic equilibrium out to a radius $R_w = 10^{11}$ cm from the center of the sun[1]. At this radius we connect the corona to a simple polytrope model of the solar wind, with a single polytrope index $\alpha$ holding all the way out to infinity.

The equation describing the isothermal part of the corona between $R_c$ and $R_w$ is simply

$$0 = -\frac{2N\mu GM_\odot}{R^2} - \frac{d}{dR}\,(2NkT_c) \tag{8}$$

where $G$ is the gravitational constant, $M_\odot$ is the mass of the sun, and $R$ is the radial distance from the sun's center. Equation (8) readily integrates to give $N_w$ in terms of $N_c$:

$$N_w = N_c \exp\left\{-\frac{GM_\odot\mu}{kT_c}\cdot\left(\frac{1}{R_c} - \frac{1}{R_w}\right)\right\} . \tag{9}$$

Our basic solar wind equations are the hydrodynamic equation

$$2N\mu v \frac{dv}{dR} = -\frac{dP}{dR} - \frac{2GM_\odot\mu N}{R^2} , \tag{10}$$

1. From here on the subscript w will refer to quantities evaluated at this radius. In selecting this radius for the base of our solar wind we are simply following Parker (1963), who found that it provided a better fit to the observed solar wind than would a lower value of $R_w$.

where $v$ is the solar wind velocity and $P$ is the pressure; the continuity equation

$$NvR^s = \text{constant} \tag{11}$$

where the matter is assumed to follow the magnetic field lines and $s$ is a parameter describing the rate of divergence of the field lines (e.g., $s = 2$ for radial field lines); the ideal gas law

$$P = 2NkT, \tag{12}$$

and a heat equation, for which we use a polytrope law

$$P \propto N^{\alpha} \tag{13}$$

where $\alpha$ is the polytropic index ($\alpha = 1$ for isothermal expansion and $\alpha = 5/3$ for adiabatic expansion). Making the substitutions

$$\psi \equiv \frac{\mu v^2}{2kT_c} , \tag{14}$$

$$\lambda \equiv \frac{GM_{\odot}\mu}{R_w kT_c} , \tag{15}$$

and

$$\zeta \equiv \frac{R}{R_w} , \tag{16}$$

we obtain our general solar wind equation

$$\left\{1 - \frac{\alpha\ \psi_w^{\frac{\alpha-1}{2}}}{2\psi^{\frac{\alpha+1}{2}}\ \zeta^{(\alpha-1)s}}\right\} \cdot \frac{d\psi}{d\zeta} = \frac{1}{\zeta}\left\{\frac{\alpha s\ \psi_w^{\frac{\alpha-1}{2}}}{\psi^{\frac{\alpha-1}{2}}\ \zeta^{(\alpha-1)s}} - \frac{\lambda}{\zeta}\right\} \tag{17}$$

for $\alpha \neq 1$. Equation (17) integrates to give

$$\psi - \frac{\lambda}{\zeta} = - \psi_w^{\frac{\alpha-1}{2}} \left( \psi^{-1/2} \zeta^{-s} \right)^{\alpha-1} \frac{\alpha}{\alpha-1} + \psi_w + \frac{\alpha}{\alpha-1} - \lambda \tag{18}$$

subject to the conditions

$$\alpha s < \lambda < \frac{\alpha}{\alpha-1} \tag{19}$$

for the existence of a subsonic-supersonic transition (as elaborated by Parker, 1963). To find the value of $\psi_w$, we use the fact that the brackets on the left-hand and right-hand sides of equation (17) must both vanish at the same critical radius $\zeta_{crit}$ (for a wind-type solution), and then solve the equations to obtain

$$\psi_{crit} = \frac{\lambda}{2s\ \zeta_{crit}} \tag{20}$$

and

$$\zeta_{crit} = \left( \frac{2}{\alpha\ \psi_w^{\frac{\alpha-1}{2}}} \right)^{2/m} \left( \frac{\lambda}{2s} \right)^{\frac{\alpha+1}{m}} \tag{21}$$

where

$$m = \alpha + 1 - 2s(\alpha - 1) \ . \tag{22}$$

On substituting (20) and (21) into (18) and assuming that $\psi_w << 1$, we finally obtain

$$\psi_w = \left( \frac{4}{\alpha^2} \right)^{\frac{1}{\alpha-1}} \left( \frac{\lambda}{2s} \right)^{2s} \left\{ \frac{\alpha - \lambda(\alpha - 1)}{m} \right\}^{\frac{m}{\alpha-1}} \tag{23}$$

and

$$\zeta_{crit} = \frac{\lambda}{2s} \frac{m}{[\alpha-\lambda(\alpha-1)]} \quad . \tag{24}$$

The particle flux $J$ at $R = R_w$ is just

$$J_w = 2N_w v_w = 2N_w \left( \frac{2kT_c}{\mu} \right)^{1/2} \psi_w^{1/2} \quad . \tag{25}$$

Hence, by continuity, the particle flux at the base of the corona $J_c$ must be

$$J_c = \left(\frac{R_w}{R_c}\right)^s \; 2N_w \left(\frac{2kT_c}{\mu}\right)^{1/2} \psi_w^{1/2} \quad . \tag{26}$$

We have only to multiply equation (26) by the energy gain per particle $E_{pp}$ between $R = R_c$ and $R = \infty$ to obtain the total flux of energy at $R = R_c$ going into the solar wind. To find $E_{pp}$, we add the gain in gravitational energy between $R = R_c$ and $R = \infty$ to the kinetic energy at $R = \infty$, found from equation (18), and subtract the thermal energy at $R = R_c$ to obtain

$$E_{pp} = kT_c \left(\frac{\alpha}{\alpha-1} - \frac{3}{2}\right) \quad . \tag{27}$$

(There is no thermal energy at infinity since, for $\alpha > 1$, T goes to zero.) Hence, using euqations (9), (23), (26), and (27), we find that the total flux $F_u$ of energy going up into the solar wind from $R = R_c$ is given by

$$F_u = kT_c \left(\frac{\alpha}{\alpha-1} - \frac{3}{2}\right)\left(\frac{R_w}{R_c}\right)^s 2N_c \exp\left[-\frac{GM_\odot\mu}{kT_c}\left(\frac{1}{R_c} - \frac{1}{R_w}\right)\right]\left(\frac{2kT_c}{\mu}\right)^{1/2}$$
$$\left(\frac{4}{\alpha^2}\right)^{\frac{1}{2(\alpha-1)}} \left(\frac{\lambda}{2s}\right)^{2s} \left\{\frac{\alpha-\lambda(\alpha-1)}{\alpha+1-2s(\alpha-1)}\right\}^{\left(\frac{\alpha+1}{\alpha-1} - 2s\right)} \tag{28}$$

which gives $F_u$ as a function of $T_c$, $N_c$, $\alpha$ and s. (The divergence defined by s need hold steady only as far as the critical radius; beyond that radius it can change without affecting $F_u$.)

(d) Relation Between $T_c$ and $N_c$

The density in the transition region and corona is affected by what goes on in the chromosphere. Since we have no good theoretical model of how the chromosphere is affected by the heat flux from the transition region, we instead use a simple relation between $T_c$ and $N_c$ based on the empirical observation that the intensity of most lines formed in the transition region is unaffected by the presence of a coronal hole (Munro and Withbroe, 1972). Because the intensity of a given line is proportional to $N^2\left(\frac{dT}{dz}\right)^{-1}$ evaluated at the temperature of formation of that line, this implies that at a given temperature in the transition region

$$N \propto \left(\frac{dT}{dz}\right)^{1/2} \tag{29}$$

or, using equations (2) and (7),

$$N \propto T_c^{5/4} \quad . \tag{30}$$

For any column of gas in hydrostatic equilibrium, reduction of the temperature at all levels beneath a certain altitude will lead to reduction of the density of gas at that altitude relative to the density at the base of the column. However, equation (30) tells us that in the transition region the density will go down in proportion to $T_c^{5/4}$ if $T_c$ decreases. Therefore, the density $N_c$ at the base of the corona must go down by at least this factor but probably not much more, so as a reasonable estimate we take

$$N_c \propto T_c^{5/4} \tag{31}$$

for our relation between $N_c$ and $T_c$. To make this into an equality we write

$$\frac{N_c}{N_{c,o}} = \left(\frac{T_c}{T_{c,o}}\right)^{5/4} \tag{32}$$

where $N_{c,o}$ and $T_{c,o}$ are the quiet-sun density and temperature at the base of the corona.

## III. Basic Procedure

We now have euqations giving $F_d$ as a function of $T_c$ (equation (7)), $F_u$ as a function of $N_c$, $T_c$, $\alpha$, and $s$ (equation (28)), and $N_c$ as a function of $T_c$ (equation (32)). What we wish to know is how $T_c$ and $N_c$ will be affected as $s$ is altered; i.e., whether a coronal hole will be produced if the rate of divergence of the field lines is significantly increased. To investigate this question, we select a value of $\alpha$ consistent with solar wind observations, put observed quiet-sun values of $T_c$ and $N_c$ into equation (32), and substitute the result into equation (28). This allows us to write $F_u$ as a function of $T_c$ and $s$ only. By conservation of energy, we can assert that

$$F_d(T_c) + F_u(T_c,s) = F \tag{33}$$

and since we have assumed $F$ (the mechanical energy input) to be constant, we have a relation between $T_c$ and $s$ which can be solved numerically for $T_c$ as a function of $s$, once we have assigned a value to $F$. To select such a value for $F$ we evaluate the left-hand side of equation (33) for normal conditions. Once we have solved for $T_c(s)$, the value of $N_c$ for any $s$ follows immediately from equation (32).

## IV. Results

We wish to compare the results of our model with those found observationally by Munro and Withbroe (1972) as given in their Table 1.

For the quiet-sun temperature and density we use their results for the quiet region adjacent to the hole (their position A) rather than their OSO-4 standard quiet region, because the latter probably contains contributions from regions with closed magnetic field configurations, which are not relevant to our model. For the value of $s$ (the magnetic field divergence rate parameter out to the critical radius defined by equation (11)) corresponding to this quiet region we take $s = 1$. This somewhat arbitrary choice is made to reflect the fact that if the field lines in the hole diverge significantly faster than radially, then the field lines in regions adjacent to the hole probably diverge somewhat slower than radially. In order to select a value of $\alpha$, we tried all values from 1.10 to 1.20 (in steps of 0.01) and calculated $v_E$, $T_E$, and $N_E$ at the earth's orbit for each case (assuming that the field lines become radial beyond the critical point). The value of $\alpha$ which gave the best fit to the observed quiet solar wind was $\alpha = 1.15$; for this case we found $v_E = 330$ km/sec, $T_E = 1.3 \times 10^5$ K, and $N_E = 5$ cm$^{-3}$. In Figure 2 we plot the resulting coronal parameters $T_c$, $N_c$, and $N_w$ as a function of $s$ for this value of $\alpha$.

The results given in Figure 2 can be compared to the parameters found by Munro and Withbroe for the center of the hole they observed (their position D): at this position they found $T_c = 1.05 \times 10^6$ K and $N_c = 3.0 \times 10^8$ cm$^{-3}$. If we consider $s = 4$ or $5$ to be a reasonable reflection of the diverging field pattern one might expect, we find our results to be a little higher than those of Munro and Withbroe but definitely in the same general neighborhood (we would even expect our $N_c$ to be a little on the high side owing to the nature of our derivation of equation (32)). In view of the simplicity of our model, the agreement lends considerable support to our original hypothesis that diverging field patterns actually cause coronal holes. The only point of substantial disagreement between our results and those of Munro and Withbroe concerns $F_d$. Their value of $F_d$ drops by a factor of about 8 (from $1.0 \times 10^6$ to $1.3 \times 10^5$ erg cm$^{-2}$ sec$^{-1}$) while ours is much lower and drops only by a factor of about 2 (from $1.7 \times 10^5$ to $7.9 \times 10^4$ erg cm$^{-2}$ sec$^{-1}$).

## V. Discussion

Before we close, there are a few points warranting further comment. One is that, while our solution seems to assume a single value of $s$ to hold all the way to infinity, both the energy flux into the solar wind and the velocity at infinity are unaffected by connecting up our solution to one with a different value (presumably $s = 2$ for radial flow) at any point beyond $\zeta_{crit}$. For the case illustrated in Figure 1, $\zeta_{crit}$ is 15 for $s = 1$ but has dropped to half that value when $s = 3$. Indeed, it is just this lowering of $\zeta_{crit}$ with increasing $s$ which provides the physical basis for our results: the lower the radius at which the expansion velocity becomes supersonic, the higher the corresponding density will be and hence the greater the flow of particles and energy into the solar wind.

One definite failing of our model is that it does not explain the observation by Krieger et al. (1973) that coronal holes are the source of high velocity streams in the solar wind. For a polytrope model, the velocity at infinity (which is approximately equal to the velocity $v_E$ at the earth's orbit) is independent of $s$ and depends only on $T_c$

and $\alpha$; this is easily seen from equation (18), noting the definitions in equations (14) - (16). However, since $T_c$ decreases as s increases, the net result is that $v_E$ above a hole decreases in our model, which contradicts the observations. The reason our model still works reasonably well in spite of this failing is the fact that in the quiet solar wind the gravitational energy per particle exceeds its kinetic energy by a factor of about 4. This means that the energy flow into the solar wind is not particularly sensitive to the solar wind velocity (except inasmuch as it affects the total particle flux). Hence this energy flow can increase substantially with increasing s, even though $v_E$ decreases. One can make plausible arguments that in reality $\alpha$ would vary as s varies and that the net result of this variation would be to increase $v_E$, but our opinion is that the present model is not sufficiently sophisticated to do more than reflect the gross energetics of the solar wind. Quantitatively reliable calculations must be based on a solar wind model incorporating transport equations rather than a polytrope approximation, and a model for the non-thermal heating of the solar corona.

A plot of $N_W$ vs. s was included in Figure 2 to emphasize the fact that the density in the corona decreases with increasing s not only because the density at its base goes down but also because the decreasing temperature reduces the scale height. Hence the density at $R_W$ ($\equiv 10^{11}$ cm), which is less than half a solar radius above the photosphere, drops by a factor of more than 3 as s goes from 1 to 5.

In conclusion, while we realize that our model is not a highly accurate representation of the solar atmosphere, we believe that our results support the existence of a causal connection between diverging field patterns and coronal holes.

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## Figure Captions

Figure 1. (a) Plot of observed electron density versus temperature for the upper transition region and lower corona (based on data taken from Allen, 1973).

(b) Plot of density versus temperature based on equation (5), where $N_b$ was adjusted to make the turnover occur at the same density as in Figure 1a.

Figure 2. Plot of $T_c$, $N_c$, and $N_w$ as a function of s.

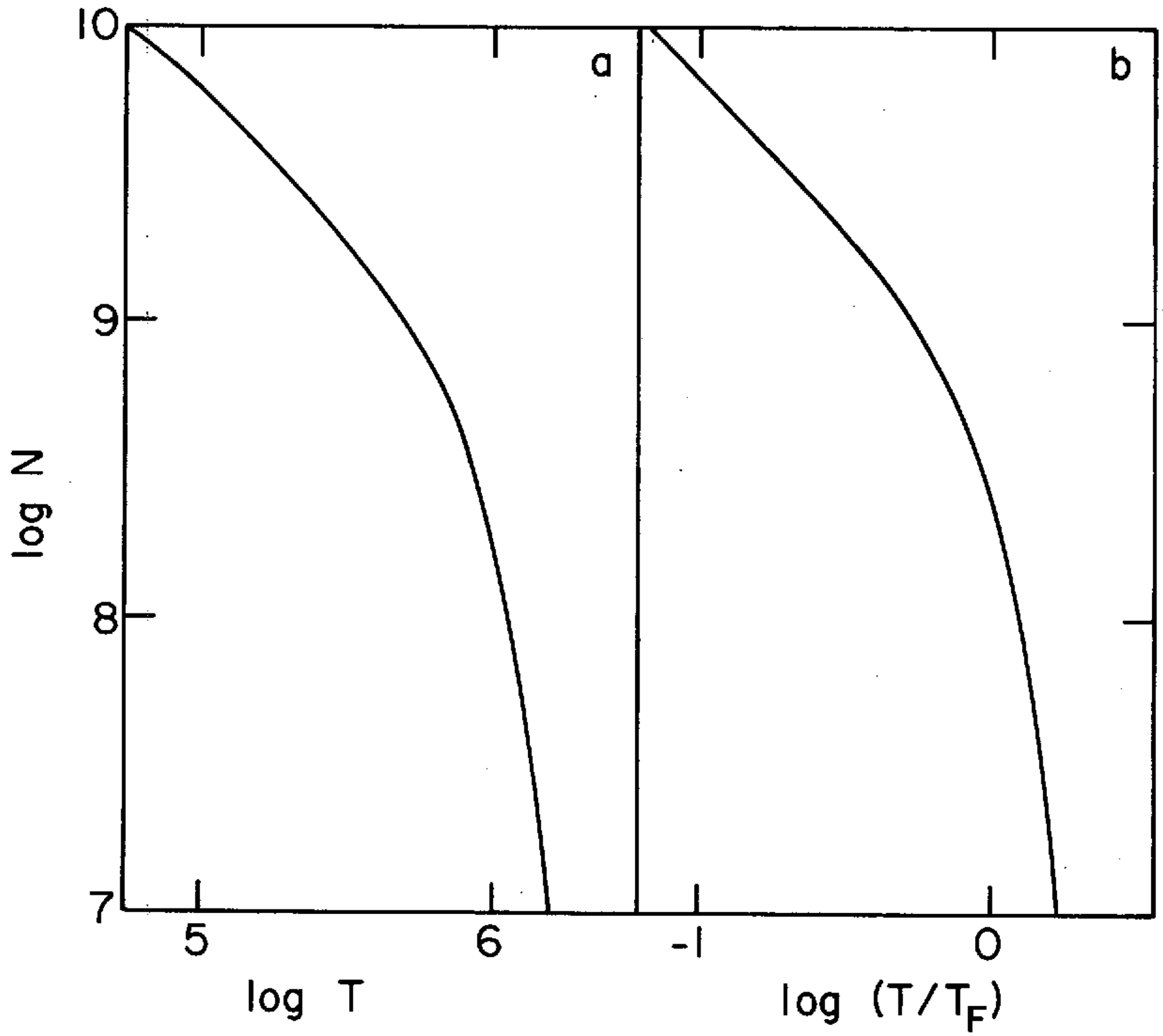
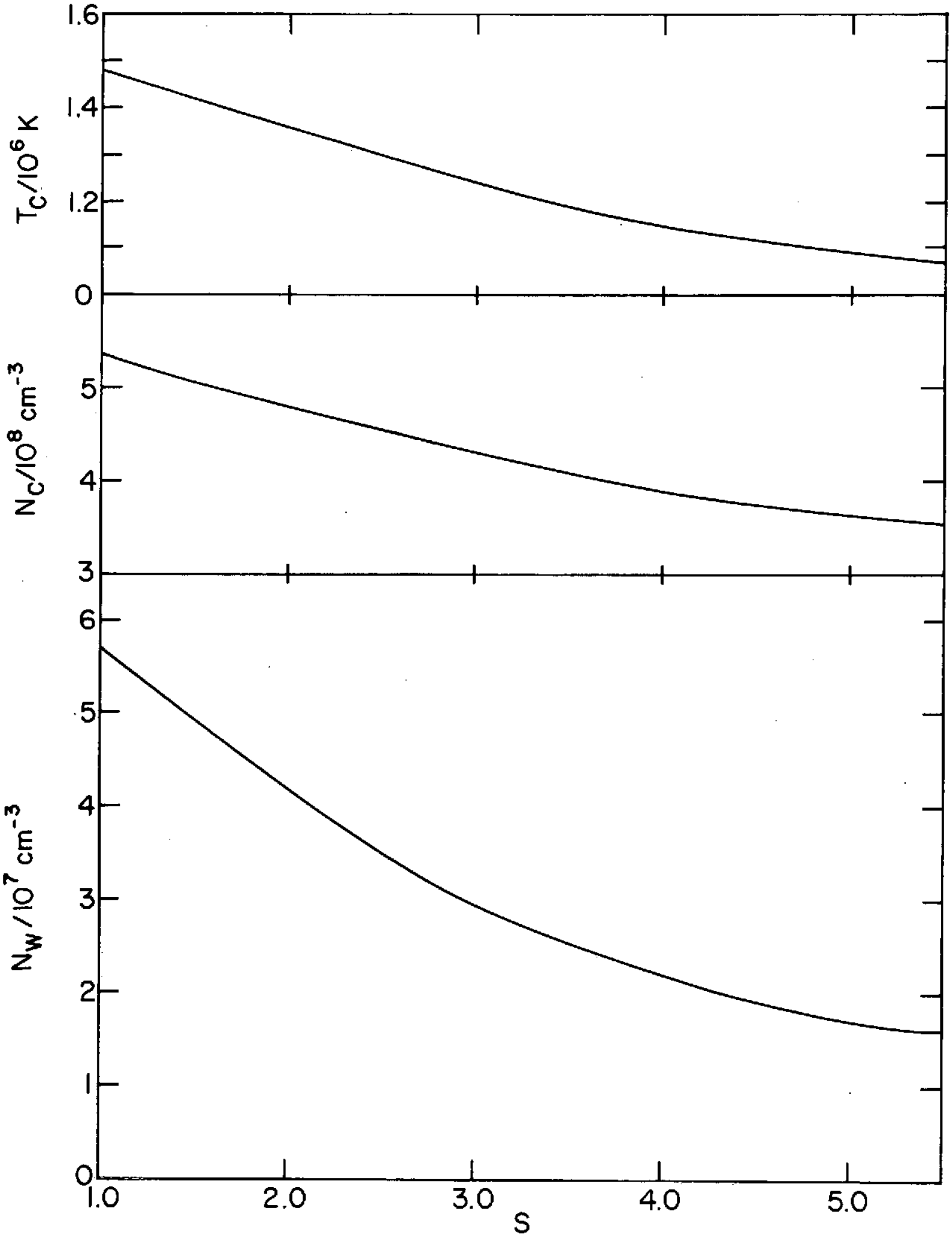



Figure 1



1.6
1.4
1.2
0
$T_C/10^6$ K
5
4
3
$N_C/10^8$ cm$^{-3}$
6
5
4
3
2
1
0
$N_W/10^7$ cm$^{-3}$
1.0
2.0
3.0
4.0
5.0
S


Figure 2